PROBLEM-BASED LEARNING AND MATHEMATICS: POSSIBLE SYNERGICAL ACTIONS

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Abstract
Starting from the assumption that problem-based learning is an instructional approach suitable for transmitting a real understanding of mathematics, we have to regret that such a methodology is not common in real teaching practice at school, as teachers usually rely on self perpetuating “traditional” methods. We would like to describe experiences conducted by the Center “matematita” (Interuniversity Research Center for the Communication and Informal Learning of Mathematics) in order to support teachers in adopting a problem-based learning approach. These actions include:

- training courses for pre-service and in-service teachers;
- organizing problem-based mathematical laboratories in school (both in primary school and at a higher level);
- designing interactive exhibitions;
- organizing web-based mathematical game contests;
- publishing books and magazines.

It is our believe that the role of University Research is crucial in carrying on synergical actions both on the teachers and on the pupils in order to improve the effectiveness of the teaching of mathematics.

Keywords
Problem-based learning, mathematics education, communication of mathematics.

1. PROBLEM-BASED LEARNING AND MATHEMATICS

Problem-based learning (PBL) is a constructivist learner-centred instructional approach based on the analysis, resolution and discussion of a given problem. It can be applied to any subject, indeed it is especially useful for the teaching of mathematics [16]. For a neat definition of PBL we refer to [27] according to which PBL “is an instructional (and curricular) learner-centered approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem”. Typically a PBL session follows these steps [15]:

- pupils are given a problem;
- they discuss the problem and/or work on the problem in small groups, collecting information useful to solve the problem;
- all the pupils gather together to compare findings and/or discuss conclusions; new problems could arise from this discussion, in this case
- pupils go back to work on the new problems, and the cycle starts again.

In spite of researches documenting the effectiveness of PBL, we have to regret that such a methodology is not common in real teaching practice at school, as teachers usually rely on self perpetuating “traditional” methods [16]. University Research should thus focus on finding ways to support teachers practice in schools. We will briefly discuss a few actions held by the Centre matematita, Interuniversity Research Centre for the Communication and Informal Learning of Mathematics, joining the Universities of Milano, Milano-Bicocca, Pisa and Trento (http://www.matematita.it/). We will also try to focus on the correlations between all these actions.

Citation information:
1.1 Teacher training

It is now a fact that Universities are in charge of pre-service teacher training in most of our countries. What we sometimes lack is any kind of link between Universities and in-service teachers, at least this is the case in Italy. This fact is especially true for primary school teacher, as in Italy the pre-service primary school teacher training university degree courses have been active since no more than a decade. In other words many of the teachers now working in school did not have any university training at all, and are now willing to have some support by the University.

When designing refresher courses for teachers, it is important to focus not only on disciplinary contents, but also on methodological aspects. That is, on one side one has to deal with the peculiar nature of mathematics, keeping in mind that ordinary people usually have a conflicting relation with mathematics. This bad attitudes is often due to wrong beliefs about the subject, which is seen as a dull set of rules to be learned by heart. Thus teachers should give their students the opportunity to see mathematics as a living subject, that is as an exploratory, dynamic, and evolving science. Teachers should teach their students that the short answer is not what they really need, and that things must be looked at in depth. And this is the place where methodology steps in: simply put, if mathematics is an exploratory subject, then it must be taught in an exploratory way. The socio-constructivist perspective enforce this point of view, and doing mathematics in a PBL learner centred environment is a chance to give students a real understanding of the subject. Once again, the basic idea is that you do not appreciate and learn mathematics if you do not do mathematics.

Moreover, once accepted that PBL is an effective way to teach mathematics, one has to act on teachers’ beliefs in order to have them actually use PBL in their teaching practice. According to the Centre matematita’s experience, the most effective way to have teacher willing to use a PBL methodology is to have they experience themselves the joy of mathematical discovery, hence the courses for in-service teachers held by the Centre now adopt PBL. By undergoing PBL, teachers acquire beliefs in favour of a PBL methodology as they see that in this way they learn some good maths [16]. There is one more aspects that needs to be stressed. In order to use PBL teacher must be trained as their role is crucial. Once more the personal experience is the best ground on which they can build an effective teaching practice.

matematita conducted such courses in various occasions, the most conspicuous one being the “Year of the Geometry”, held in collaboration with the Sovrintendenza scolastica in Bolzano [23]. The kind of problems proposed in such courses varies, according to the subject and to the level. Typically problems must be engaging, not of immediate solution and usually rely on disciplinary competences superior to the one that the teachers are supposed to teach (“the first rule of teaching is to know what you are supposed to teach; the second rule of teaching is to know a little more than what you are supposed to teach” [24]). The final discussion often focuses on pedagogical implications of the themes dealt with. We give a purely indicative example of a problem usually discussed with primary school teachers (for a discussion of the primary school mathematics involved with this problem e.g. see [13]):

On a geoboard (or equivalently on squared paper) draw as much as possible rhombi whose sizes are 5 units long.

These courses allowed matematita to build a network of connections with in-service teachers, useful to collect their objections to this kind of activities. The main criticism concerns the fact that PBL is time consuming, and it is thus important to show teachers that the time is well spent as pupils get a real understanding of the subject, and gain at the same time many collateral competencies [20]. A further
objection coming from the teacher is that such activities rely on “good” problems (e.g. see [6, 20, 27]) and in-service teachers do not feel to be competent enough to create suitable ones. Researchers can provide teachers with ready to use material, and we will show some examples by matematita in the next sections.

1.2 Web based game contests

In 2002, as part of a course for in-service primary school teachers a game contest for school children was set up. The teachers, with the support of the researchers of the Dipartimento di Matematica ‘F. Enriques’ of the Università degli Studi di Milano, created a set of problems to be proposed to the children. The game contest was in fact just an excuse to set up a PBL environments: children were asked to work in small groups on the given problems with (almost) no time constriction, to write down their solutions and to mail them to the “Redazione” that gave them feedback (for a description of the experience see [10]). The teachers acted as mediators between the pupils and the university researchers.

The game contest has been re-proposed yearly and has been extended to secondary school. The researchers are now in charge of creating the problems, while teachers set up the class environment. Since 2005 all contacts between teachers and researchers pass through a web site: teachers register their classes to the site, collect the problems on certain dates together with information on the mathematical topics involved and suggestions on how to conduct the final discussion; they finally upload their pupils’ answers to the site. Also an online private forum is available. Due to the kind of activity the answers are manually analysed by the researchers as the pupils are asked to give full explanation of their solutions. If the proposal will be extended to high school it might be possible to set up an automatic analysis of answers, e.g. using a software like WIMS [28]. The web site allows accounting of the classes involved. In brief, 790 classes registered in 2005, 685 in 2006, and 555 in 2008 (in spite of the interruption). What emerges is an high rate of teachers that keep “returning” and proposing the games to their classes, year after year.

The school principal decided that the walls have to be decorated with a long line of tiles. To save money he agreed with the teachers that pupils are going to mould all the tiles the school needs. Then they will ask parents to burn them in the oven collecting the tiles ten by ten in stripes consisting of 4 tiles with the sun, 3 with the moon and 3 with a star, like the one we draw right here:

(we put a small number near each tile).

Fig. 2: http://www.quadernoaquadretti.it/giochi/

The problems proposed, in order to be engaging and of no immediate solution, are mainly based on “classical” mathematical problems and/or cover part of the syllabus (modular arithmetics, combinatorics, area, volume, probability, hints on topology, . . . ). An example is given in Fig. 2 and more are available on the web site where all the problems are collected in a public repository for the use of any teacher willing to propose them to their own classes. Some of the problems have also be published in [8]. Classroom activities based on these problems are documented in various degree thesis (e.g. see [1, 9, 11, 25, 26]).

1.3 Problem-based mathematical laboratories

The activities described so far are conducted by the teachers in their classes. As soon as the school level increase a more active role of the university researchers might be required. This role was also recognised by the Italian Ministry of Education, that suggested university researchers should offer “mathematical laboratories” for high school and financed a project (Progetto lauree scientifiche, 2005-2009). matematita offers a series of “laboratories” based on a PBL model on various arguments: cryptography, polyhedra, mathematical modelling, topology, optimisation, and 4-dimensional geometry. The Centre
has also experienced laboratories outside of the class context. The cooperation with the Sovrintendenza scolastica in Bolzano is carried on with the “Bottega” (apprenticeship): selected groups of students of different schools of Trentino-Alto Adige spend a week in a pleasant spot working on mathematical problems under the supervision of university researchers (this experience is described in [21]). Similar more “informal” experiences have been conducted with the whole classes as a side of their instructional trips. The mathematical exhibition we will describe in the next section offer a valid setup for informal mathematical experiences too.

1.4 Interactive exhibitions

One of the field of research of matematita is the design of mathematical interactive exhibitions, trying to connect popularization of scienze with teaching.

matematita experience is sited in the horizon of numerous experiences of this kind, where the two milestone stones are L’occhio di Horus. Itinerari nell’immaginario matematico, by Michele Emmer (1989) and Oltre il compasso (Beyond the compass, 1992), with the idea of a museum for mathematics (e.g. see [19]). “Thought for a non specialist public, Oltre il compasso had the ambition to illustrate examples of maths facts hidden in our everyday life [...] The public would have left the exhibition having learned something, having guessed something more, and having gained interest on something else. [...] Mathematics can be told, with obvious expedients, to whoever gets over his/her initial inhibition and follow his/her own curiosity. Which such an aim, it turned out that the winning move is to involve as much as possible the visitor into the mechanism of the objects”, not simply by pressing a button, as “you learn math with your head, but also with your eyes and with your hands”. If we share this idea of presenting maths through interactive exhibits, we have to face the problem of actually constructing such exhibits: “the making of the exhibits seems to be a simple and natural thing, almost obvious, but this is not the case and you can build successful exhibit only with a strong mathematical experience” [19]. We thus need to find branches of mathematics and problems that can easily be told to the general public. The first example of exhibition designed by the researcher of matematita is Simmetria, giochi di specchi (Symmetry, playing with mirrors), in which visitors explore the world of friezes and wallpaper patterns through mirror boxes (for a description see [2]). Simmetria, giochi di specchi is permanently held at the Dipartimento di Matematica ‘F. Enriques’ of the Università degli Studi di Milano. This location is allowing a monitoring of the visitors’ reactions, trying to hit a change in their attitude towards the subject. Visitors’ involvement in the exhibits starts with a problem in a PBL view. “An experience of this kind offers nothing but a behaviour and a working method that we believe is useful” Maria Dedò says in [18], and goes on “all the different kinds of visitors pass through four different steps [...] It is indeed surprising the fact that these steps can be identified for every category of visitors, from elementary school kids, through high school students, up to their teachers, adults with any level of education”. These stages are well described in [18]: “1. The first reaction is wonder; 2. secondly the visitor get involved and tries to ≪do≫ things,
and not simply observe things made by others; 3. afterwards they start asking questions, wondering "why" the things they have observed worked that way; 4. finally they try to answer these questions". What Maria Dedò points out is that "the crucial step is the one from the first to the second: it is not automatic and sometimes the passage from "watching" to "doing" fails, [...] while, for every category of public, the passage from the second to the third and from the third to the fourth steps turn out to be indeed natural (provided each one is given its own timing)". As Di Sieno, Dedò too stresses on the choice of the subject "the depth of the subject is what makes the same objects and the same phenomena useful to illustrate mathematical facts at different levels [...] the different viewpoints make it possible for everyone to get experience at their own level of maturity". On this foundations subsequent exhibition were build, the most noticeable being matemilano and matetrentino. The aim of exhibitions of this kind is to illustrate mathematical problems to the visitor in a PBL view. First the problem is stated in a poster (possibly in a simplified form) and visitors are asked to solve it, using the tools provided. As we had visitors of many levels (from elementary school to university professors and researchers) the problem had to be stated in different forms: a simplified form for the younger, but still we wanted the higher level visitors to find the "correct" statement of the problem. Let's look at an example from matemilano: we start from the shape of the map of the city of Milan. The shape of the city is indeed quite common: many cities in the world have an almost circular shape. What we find out is that there is a mathematical reason for this: the mathematical problem behind human architecture can be stated as follows: "among all the possible city shapes which one allows the biggest area with the shortest walls?" Given a flexible wall,

![Fig. 4: Making the city bigger](image)

visitors are invited to experience and asked to fill in the city, to make it "bigger", and to look at the shape it assumes (see Fig. 4). They find out the “natural” shape is a circular one. The “young” visitor is thus lead to an “intuitive” answer to the question. But still the general mathematical problem is stated "among all the plane figure of a given area which one has the shortest perimeter?". The solution of the general problem is visually given in a poster, and all the mathematical details can be found in a leaflet next to the exhibit.

The exhibition matemilano was held at the Museo Nazionale della Scienza e della Tecnologia “Leonardo da Vinci” (Milano) starting from September 2003, accompanying the XVII Congress of the Unione Matematica Italiana (Italian Mathematical Union). Originally scheduled until February 2004, its enormous success urged the museum to extend it until the end of May 2004. In matemilano the city of Milan becomes the “excuse” to talk about mathematics. The exhibition is built as a path divided into four sections (Maximum and minimum, Visualization, Topology, and Symmetry) and counts roughly 27 exhibits, each presenting a different problem (we refer to the exhibition catalog [3] and to [14] for a full description). The museum estimates that matemilano had around 50,000 visitors. Moreover, matemilano was the first experience in Italy with a “virtual” section: the modern technologies were used to help the visi-
tors to complete their tours. Not all the mathematical problems presented could be solved “with hands”, but the visitor had to use his/her imagination to figure out the solution. This is the point where a “virtual” counterpart to the “real” exhibit could help (the virtual exhibits are available in CD-ROMs, see [4, 12]). matemilano was followed by matetrentino in which the city of Trento and its surrounding were taken as the underlying theme. The exhibition matetrentino was held at the Museo Tridentino di Scienze Naturali (Trento), from February 2006, for about a year (see also the catalog [22]). This second event in Trento shows once more that Milano was only an “excuse” to talk about mathematics, but the fact that the visitors had to face with familiar places made easier for them to get involved with the exercise of abstraction that mathematics requires (it was surprising to discover that people were visiting matemilano to watch their city from a different perspective, to re-discover sights of the city they often overlooked due to the hurry of their lives). The format “mathematical exploration of the cities” seems to be responding to the recent common wish for mathematics and we strongly recommend others to reuse this format for their own town. Hopefully many other localities could give their inhabitants the occasion to see their own environment “with different eyes” [5].

1.5 Publishing

All the activities described so far inspired a publishing activity.

![Image 5: The Quaderni a quadretti](image)

The teacher training courses gave stimulus to the series “Quaderni a quadretti” (Squared paper exercise-books, see Fig. 5 and the cited [6, 8, 10, 13]), that is a collection of mathematical textbook for teachers in which disciplinary contents and teaching practice are dealt simultaneously. Some of them consists of two different parts, the “guide” for the teacher (e.g. see [8]), and the textbook for the pupils (e.g. see [7]). Interactive exhibitions had as a side effect the publishing of the catalogues and CDs (see Fig. 3 and the cited [2, 3, 4, 5, 12, 22]).

All these publications are reach in images, and matematita’s collection of images was a stimulus to set up a project of iconographic reference: Immagini per la matematica (Images for mathematics, [17]). Right from its founding, the Centre has had a vast collection of images (several thousand) and animations (several hundred); the idea behind Images for mathematics is to make this collection of images and animations available by creating a tool to communicate mathematics ideas. In today’s society the key role played by communication through images is universally recognized. However, there are still many areas in which the potential of this type of communication is not exploited to the full, or rather
than highlighting the scientific content, the images get overpowered by special effects. What is needed for those who operate in the field of communication is to have tools that provide the right balance between contents and images. matematita is particularly interested in this problem since communication through images is by nature not only an informal way for imparting information but also for learning it. The tool is designed to be user-friendly while still ensuring a high level of scientific correctness alongside top quality relevant images. The archive is continually evolving and can be consulted online (http://www.matematita.it/materiale/).

Finally the Centre is publishing a mathematical magazine for high school students. Initially thought as a translation of the French analogue, the magazine Xlatangente discusses various mathematical themes and promotes interaction with the readers through a web site, hoping to give students and teachers stimuli for “doing” mathematics.

References


