PROBLEM-BASED LEARNING AND TEACHER TRAINING IN MATHEMATICS: HOW TO DESIGN A MATH LABORATORY

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Abstract
A long tradition of educational research has promoted the use of active learner-centered methodologies at school, especially with the use of pedagogical laboratories. When mathematics is concerned, it seems that PBL (Problem-Based learning) is the most suitable approach, for a more effective long-life learning. Nevertheless, these recommendations do not seem to find their proper location in school practice, where self-perpetuating “traditional” methods impose themselves.
In order to discourage the misuse of such so called traditional methods, many authors observe that teacher training should adopt, at least in part, the exact methods that are pleaded in the lectures. This very notion inspires the teacher training program at the University of Milano-Bicocca, during which prospective primary school teachers are required to attend PBL laboratories, that accompany the “theoretical” discussion on the potential of this instructional method.
This contribution will present the structure of one of these laboratories and the achievements obtained by the students.
The laboratory we will describe is specifically aimed at stimulating in the students the understanding, through their experience, of how PBL really works and to train them to design PBL activities they could use in a math laboratory once they become teachers.
A preliminary qualitative analysis of the final essays of 68 students attending the laboratory will be provided with particular attention to:
• the ability to observe the activation of their own mathematical knowledge by mean of the PBL approach;
• the ability to transpose the lab activities, without distorting their essential mathematical content, to a level suitable for primary school children.

Keywords
Problem-based learning, mathematics education, pre-service teacher training.

1. PROBLEM-BASED LEARNING AND MATHEMATICS
Research in mathematics education is universally aware of the key role of active methodologies for an effective teaching and learning of mathematics. In an ideal learning path, students should have the opportunity to experiment and apply their mathematical knowledge with the help of laboratories, alongside traditional lectures, in which the students work actively on problems. Such problem solving activities, if corresponding to the framework that can be rooted back as the early works of the mathematician Polya, allow the pupils to develop a “mathematical thinking” (e.g. see [10, p.339]), and give them the opportunity to see mathematics for what it really is: “an exploratory, dynamic, evolving discipline rather than […] a rigid, absolute, closed body of laws to be memorized” [6, p. 84]. If you can lead pupils to “do math”, they gain a better grasp on the mathematical notions with the effect of a longer lasting knowledge (e.g. see [1], [7], [9]).
From the educational sciences point of view, this tradition of mathematical problem solving derived from Polya’s works can be situated in the problem-based learning (PBL) methodology, somehow making clearer the meaning of the term, as it is well known that the term problem solving is often used in contradictory ways, e.g. see [10, p. 335]. We refer to [8] for a neat definition of problem-based learning: “PBL is an instructional learner-centered approach that empowers learners to conduct research, integrate
theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem”. Typically, a PBL activity is organized according to these steps (e.g. see [2] or [3]):

- pupils are given a problem;
- they discuss the problem and/or work on the problem in small groups, collecting information useful to solve the problem;
- all the pupils gather together to compare findings and/or discuss conclusions; new problems could arise from this discussion, in this case
- pupils go back to work on the new problems, and the cycle starts again.

The validity of PBL as an instructional method has been widely studied and many researchers confirm its effectiveness in many fields (for a review e.g. see [5], [8]).

1.1 PBL and teacher training

In spite of researches documenting the effectiveness of active learning, we have to regret that such methodologies are not common in real teaching practice at school, as teachers usually rely on self-perpetuating “traditional” methods, especially in mathematics (e.g. see [10, p. 360]). It has therefore long been hypothesized that the introduction of active learning (and in particular of problem solving/PBL activities) in the training of prospective teachers is necessary to reverse this trend: “A teacher who acquired whatever he knows in mathematics purely receptively can hardly promote the active learning of his students” [7, vol. 2, p. 113].

We will describe the math activities planned for the pre-service primary school teacher program at the University of Milano-Bicocca. Since 2011, according to the guidelines set by the Italian law, prospective teachers have to attend a five year long combined Bachelor and Master degree. This degree program includes three modules in mathematics:

- **Foundations of mathematics** (year 1): lectures focus mainly on purely mathematical contents (numbers, arithmetic and probability), while example classes are devoted to the explanation and resolution of standard math exercises;
- **Basic mathematics for teaching** (year 2): lectures focus on mathematical contents (measure and geometry), and, as above, example classes are devoted to standard exercises; moreover, students are required to attend a compulsory laboratory in which active methodologies are shown in action (students experience learning in a lab);
- **Mathematics teaching** (year 4): lectures focus mainly on methodologies (active learning and PBL) and teaching examples (analysis of problems); moreover, students are required to attend a compulsory laboratory in which a further analysis of teaching methodologies is carried on (prospective teachers experience teaching through a lab).

The progression of such modules clearly reflects the idea that a good teaching of mathematics can only be built on solid disciplinary contents, with the gradual transition from purely content-related topics to explicit references to methodologies. The degree program includes numerous pedagogical modules which cover instructional methods and recommend the idea of active learning.

In the next section we will describe an example of laboratory related to the course “Mathematics teaching”. Due to the large number of students (about 300), the class is split in small groups (usually of about 30 pupils) and the actual content dealt in the laboratory attended by each group can vary. The common aim of all the laboratories is to have a session during which prospective teachers have to design some teaching activity.

2. AN EXAMPLE OF A LABORATORY

The laboratory object of analysis had as its main theme probability and the teaching of probability; it was also aimed at having the students experience PBL. The starting point for a PBL activity is a suitable “problem” given to the students to solve: the more the problem is “ill-structured”, the more the activity can be effective [8, p. 15]. In this case pupils were given the rules of a modified version of the well-known Yahtzee game and the “problem” for them was to find the best strategies to win the game.

The laboratory was organized in 3 sessions of 4 hours each. More specifically:

- **Day 1:** learning the game
  - short introduction and explication of the aims of the laboratory (students are told that the aim of the activity is to “learn how to create a math laboratory”) (5 mins);
– brainstorming on the word “game” (25 mins);
– group game (round 1): students are divided into groups, each group is given a written copy of the rules of the game and a set of dice, groups are asked to play the game according to the given rules (1 hour);
– discussion in large group on the activity so far, comparison between the different interpretations of the rules by the different groups and how these different interpretations have in fact determined substantially different games (30 mins);
– group game (round 2): a second game with a deeper awareness of the rules after the previous discussion (1 hour)
– final discussion in large group based on the question “which math did you do?”, aimed at stimulating a reflection on the mathematical contents touched during the game (1 hour).

At the end of the session, the tutor assigned a short group essay (to be handed in via Moodle) based on the question “what are the strategies of victory that you have identified?” and announced that such strategies would have been used to play a “group against group” tournament in the following session.

• Day 2: the tournament
  – group against group game (2 hours and 30 mins);
  – class discussion about the game (and about strategies and probability facts) (40 mins);
  – class discussion about the role of the teacher (50 mins).

• Day 3: laboratory activity design
  – group work: groups are asked to design activities for pupils (in the range K-10)
  – class discussion: discussion on the groups’ projects.

At the end of the laboratory, students were asked a final report with a personal review of the experience (and these reports will be the basis on which we will build the analysis in the next section). We believe it is useful to highlight the following aspects that have emerged during the laboratory.

• The activity has been proposed twice, to two distinct group of 42 students each, so we will in fact speak about “two” laboratories.
• In the initial brainstorming about the word “game”, the instrumental use of games in teaching practice emerged predominantly, rather than the pure playful meaning of the term (probably this was influenced by some implicit “didactical contract”: the students expected the tutor to expect such answers); the tutor had to lead the conversation to stimulate more sincere answers (“when was the last time that you played a game?”) and announced that the laboratory would have been experience of playing a game for its own sake.
• The rules of the game were distributed on paper without any prior discussion, so that the task of not only “reading and understanding”, but also “sharing the game rules” was implicit; the name of the game was not communicated to the students (and none of the students explicitly said that they already knew the game); the idea that this “building and sharing” the rules was indeed a mathematical activity (e.g. see [4, p. 12]) emerged in the final discussion of the day.
• We choose to use a modified version of the standard Yahtzee game. These are the changes with respect to the rules one can find e.g. on Wikipedia [11]:
  – the bonus on the upper section has been lowered to 60 (so to allow for more “backup strategies”);
  – the lower section has been narrowed to only 4 categories: straight (corresponding to the “Large straight” on the Wikipedia page, 20 points), full house (30 points), four of a kind (40 points), five of a kind (50 points);
  – a middle section has been added with a very peculiar rule: two lines (marked + and −) are added in this section, for both line the sum of all dices is taken (similarly to the “chance” row), the value in the − line is subtracted from the value in the + line and the result is multiplied by the value in the line of 1 (the first line in the upper section);
  – the total number of rounds is thus 12.

The aim of these modifications was discussed at the end of the session. The tutor brought out the idea that a teacher who designs an activity can change whatsoever rules according to different didactic purposes.
• Due to the high number of students attending the lab (1 tutor every 42 students i.e. 7 groups), tutoring of group work was minimal. The tutor just monitored group work with nearly no direct
action: the class discussions were instead used to give suggestions or corrections (the need of "corrections" arose from the confrontation between the groups).

• The main divergence on the interpretation of the game rules was precisely for the rule that characterizes the game: once the round is over, the player must choose which row of the table to fill in and this is what activates, even if only at an informal level, probabilistic knowledge. Some groups played as if the table had to be filled consecutively, from the first row to the last. The discussion brought out the idea that in this way the game was less entertaining, because it was based more on chance than on strategy.

• Metacognitive discussions ("which math did you do?", "what was the role of the tutor?") aroused two different reactions in the students: on one side surprised the students and on the other involved them very much. These discussions made explicit the didactical core of the lab.

• During the tournament, it was decided that each member of the group had to play singularly a round, with almost no suggestions from the rest of the group (there were 12 rounds to be played and 6 members in each group, so every student had to play 2 rounds). This was clearly an imposed instrumental forcing, aimed at allowing the tutor to verify how much the individual students had internalized the strategies, previously discussed during group work. This choice was analysed afterwards in the discussion about the role of the teacher. Due to the large number of groups this phase turned out to be very long (two and a half hours) and not all of the students managed to keep a constant level of attention for all the time.

• In both laboratories, the tournament was won by a group that had previously elaborated a rich variety of strategies. In the discussion after the tournament, many questions emerged on probability facts (the 40 minutes devoted to this phase turned out to be insufficient).

3. ANALYSIS OF STUDENTS’ REACTIONS

At the end of the laboratory students were asked to compile a written report on the lab experience, providing them with a series of open questions as a guideline. We primarily focused our analysis on examining whether pupils showed in their reports the ability to observe the activation of their own mathematical knowledge. In order to do so we isolated and highlighted the students’ answers to the following questions:

• what were the aims of the laboratory?
• what do you think you have learned?

We classified the answers related to “knowledge” into two only main macro-categories: knowledge about “methodology” and knowledge about “subject matter”. More precisely, regarding the question about the aims of the laboratory, we categorized as knowledge about “methodology” answers like “the aim of the lab was to . . .”

• . . . have us experience active learning,
• . . . have us experience PBL,
• . . . have us experience game-based learning,
• . . . have us learn how to design a laboratory,

while we categorized as knowledge about “subject matter” answers like “the aim of the lab was to . . .”

• . . . have us “do math”,
• . . . build mathematical knowledge,
• . . . teach probability.

For what concerns the question about the things learned, we categorized as knowledge about “methodology” answers like “I’ve . . .”

• . . . learned PBL,
• . . . learned that mathematics can be taught in a different way,
• . . . learned that mathematics can be taught through a game,
• . . . learned that mathematics can be fun,
• . . . learned how to design a math laboratory,
• . . . learned how to act as a teacher in a math laboratory,
• . . . learned how to deal with error (as a teacher),
Aims of the lab

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<th>things learned</th>
<th>method</th>
<th>method &amp; subject m.</th>
<th>subject m.</th>
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<td>method</td>
<td>15 (7/8)</td>
<td>7 (4/3)</td>
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Table 1: Knowledge activated

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Table 2: Knowledge activated (by group)

while we categorized as knowledge about “subject matter” answers like “I’ve . . .”

- . . . learned/revised facts about probability,
- . . . learned/revised facts about other mathematical domains,
- . . . learned how to bring out hidden mathematical facts,
- . . . got the stimulus to revise some mathematical facts,
- . . . learned a new game.

A total of 68 reports (34 for each laboratory) were handed in and analyzed, and the result of our analysis is summarized in Table 1. The clear majority of the students (65 over 68) gave answers related to knowledge about “methodology”. Almost half of them (30 over 68) gave only answers related to knowledge about “methodology”, that is these students did not seem to have been able to observe an activation of their own mathematical knowledge (at least not explicitly). A very few minority (3 over 68) gave answers prevalently related to knowledge about “subject matter”. In Table 2, we can notice a discrepancy between the two laboratories: in the second lab a larger part (19 over 34) gave only answers related to knowledge about “methodology”. That is, in this lab, the mathematical goals have been perceived less. With respect to the activities designed by the pupils in the final session, and our second research question, about the ability to transpose the lab activities without distorting the essential mathematical content, we have a very partial qualitative analysis. Nevertheless, a clear distinction was observed between the two laboratories. In the first one, most groups did design activities on probability, with features derived from the game experienced in the previous sessions (typically the idea that in every round the player can rolls dice three times and can choose dices to be kept and dice to be rolled again). In the second one, only one group presented an activity on probability (that is most of the groups presented activities in which chance prevailed over the aspect of probabilistic reasoning, in spite of the fact that this distinction had emerged in a previous discussion) and no elements from the Yahtzee game were used. The fact that most of the activities were not “really about probability” emerged and was extensively discussed at the end of the session.

Further analysis is required to investigate this discrepancy between the two laboratories, and to investigate whether the ability to recognize one’s own activation of mathematical knowledge is indeed related to the ability to design mathematical activities on a given subject. It would also be interesting to analyze the actual mathematical proficiency of the students: students that started the lab with a better knowledge of probability might not actually have had any “activation” of new knowledge about probability, and thus did not feel the need to mention it in their reports.

CONCLUSIONS

According to their reports, the laboratory is seen by all the students as an indispensable complementary part of the lectures, and all of them claim that, after this experience, they will adopt the methodologies used in the laboratory when they will be teachers. This could be a very promising prospect, but of course we need to be very careful. We can only hope that the regular introduction of PBL activities into the curriculum of prospective teachers can have positive effects in the long run.
References


