Relationships between eigen-vortical-axis line and vorticity line

K. Nakayama¹, H. Hasegawa²

¹ Department of Mechanical Engineering, Aichi Institute of Technology, Aichi, Japan
² Division of Mechanical Engineering, Graduate School of Engineering, Aichi Institute of Technology, Aichi, Japan

Summary

Eigen-vortical-axis line has been proposed (Nakayama et al. (2015) AIP Proc.) as a vortical axis derived from the invariant local flow topology, and shown that it follows a vortical core region whereas the vorticity line may deviate from it. The present study shows that their difference is derived from the vorticity components parallel to the swirl plane and that these components incline both lines to the swirl plane and may deviate the vorticity line.

1 Eigen-vortical-axis line and its feature

In the velocity field, the local flow around a point can be expressed by the velocity gradient tensor \( \nabla \mathbf{v} \), thus the local flow topology is specified by the eigenvalues and eigenvectors of \( \nabla \mathbf{v} \) [1]. If \( \nabla \mathbf{v} \) has a pair of complex conjugate eigenvalues \( \varepsilon_R \pm i\psi \) (\( \psi > 0 \)) and a real one \( \varepsilon_a \), and their respective eigenvectors \( \xi_{pl} \pm i\eta_{pl} \) and \( \zeta \), then the flow trajectory can be represented as \( \mathbf{x} = 2e^{\varepsilon_R t}\{ \xi_{pl} \cos(\psi t) - \eta_{pl} \sin(\psi t) \} + e^{\varepsilon_a t}\zeta \), where the local flow swirls in the swirl plane \( \mathcal{P} \) defined by \( \xi_{pl} \) and \( \eta_{pl} \), with converging or diverging according to \( \varepsilon_R \), and proceeds (or approaches) along a vortical axis \( \zeta \). Then \( \zeta = [\zeta_i] \) \( (i = 1, 2, 3) \) represents the local axis direction of the vortical flow, and eigen-vortical-axis line is defined as an axis line in which an axis point \( \mathbf{a} = [a_i] \) \( (i = 1, 2, 3) \) is derived from the following equation [2]:

\[
\frac{d\alpha_1}{\zeta_1} = \frac{d\alpha_2}{\zeta_2} = \frac{d\alpha_3}{\zeta_3}.
\]

\( \nabla \mathbf{v} = [\partial \mathbf{v}_i / \partial x_j = [a_{ij}] \) \( (i, j = 1, 2, 3) \) in an orthonormal coordinate system \( x_i \) with unit bases \( \mathbf{e}_i \) where \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \) are parallel to \( \xi_{pl} \) and \( \eta_{pl} \), respectively, can be expressed as:

\[
\nabla \mathbf{v} = \begin{bmatrix}
\varepsilon_R & c\psi & a_{13} \\
-\psi/c & \varepsilon_R & a_{23} \\
0 & 0 & \varepsilon_a
\end{bmatrix},
\]

where \( c = |\xi_{pl}| / |\eta_{pl}| \) given by the eigenequations of \( \nabla \mathbf{v} \). \( c \) specifies the vortical flow symmetry in \( \mathcal{P} \) [3]. \( a_{13} \) and \( a_{23} \) are associated with the vorticity component \( \mathbf{w} = [\omega_i] \) \( (i = 1, 2, 3) \), i.e., \( \omega_1 = -a_{23} = (\mathbf{w}, \mathbf{e}_1) \) and \( \omega_2 = a_{13} = (\mathbf{w}, \mathbf{e}_2) \). Note that \( \omega_3 = (c + 1/c)\varphi \), and \( \omega_3 \) is equal to the eigen helicity density [4] that is the vorticity component normal to \( \mathcal{P} \) [3]. \( \zeta \) is expressed as \( \zeta \parallel (-c\psi\omega_1 - \varepsilon\omega_2, c\omega_1 - \psi\omega_2/c, \varepsilon^2 + \psi^2) \) where \( \varepsilon = \varepsilon_R - \varepsilon_a \). \( \zeta \) is not orthogonal to \( \mathcal{P} \) in general, and the angle \( \theta \) between \( \zeta \) and \( \mathbf{e}_3 \) (normal vector of \( \mathcal{P} \)) is influenced by \( \omega_1 \) and \( \omega_2 \). If \( \omega_1 = \omega_2 = 0 \), then \( \zeta \perp \mathcal{P} \) and \( \zeta \parallel \mathbf{w} \).

2 Numerical Analysis

The eigen-vortical-axis and vorticity lines in the vortical region are analyzed in isotropic homogeneous decaying turbulence in Direct Numerical Simulation by the pseudo-spectral method with the phase shifting method and the wavenumber \( |k| < 121 \) [2, 5]. Figure 1 shows the vortical regions (in subregion of the analytical domain) specified by the swirling \( \varphi \) [3] that represents the swirling intensity in terms of the geometrical mean and is equal to \( \psi \) in the vortical region. \( \varphi \) is nondimensionalized by its root mean square value at the corresponding time, and Taylor Reynolds number.
Fig. 1: Vortical regions represented by contours of $\varphi = 2$, and an eigen-vortical-axis line (bold line) and two vorticity lines (narrow lines) in $37\eta \times 37\eta \times 37\eta$ domain ($\eta$: Kolmogorov length; $\eta = 0.012$). The arrows indicate the initial points of tracing the lines. The colors of the lines show (a) $\theta$ and (b) $\varphi$ in axes (denoted as $\theta_a$ and $\varphi_a$, respectively).

Re$_\lambda = 35$. An eigen-vortical-axis line and two vorticity lines are also shown in Fig. 1 that are traced with the initial points in the vortical region indicated by the two arrows (A and B). The eigen-vortical-axis lines traced from two initial points have the same orbit, and follow the vortical region with strong $\varphi$, where the vorticity and eigen helicity density have the similar features. On the other hand, one vorticity line traced from the initial point A with a large $\theta$ as shown in Fig. 1 (a) deviates from the vortical core region (contour) and has weaker $\varphi$. However, another vorticity line traced from the point B with a small $\theta$ follows the vortical region. $\omega_1$ and $\omega_2$ incline both $\zeta$ and $\omega$ to $\mathcal{P}$, and they may effect deviation from the vortical region. But a vorticity line that passes a point in the vortical core region with a small $\theta$, i.e., with small $\omega_1$ and $\omega_2$ values, seems to follow the vortical core region.

3 Conclusion

The eigen-vortical-axis line is influenced by the vorticity components parallel to the swirl plane, and they derive the difference between the axis and vorticity line. They incline the both lines to the swirl plane, and may deviate the vorticity line from the vortical core region.

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References


